Cooling of a cup of coffee

The nature of the energy transfer from the hot water in a cup of coffee to the surrounding air is complicated and, in general, involves the mechanisms of convection, radiation, evaporation, and conduction. However, if the temperature difference between the water and its surroundings is not too large, the rate of change of the temperature of the water may be assumed to be proportional to the temperature difference. We can formulate this statement more precisely in terms of a differential equation:

$$\frac{dT}{dt} = -r\,(T - T_s), \tag{3.21}$$

where T is the temperature of the water, T_s is the temperature of its surroundings, and r is the cooling constant. The minus sign in (3.21) implies that if $T > T_s$, the temperature of the water will decrease with time. The value of the cooling constant r depends on the heat transfer mechanism, the contact area with the surroundings, and the thermal properties of the water. The relation (3.21) is sometimes known as Newton's law of cooling, even though the relation is only approximate, and Newton did not express the rate of cooling in this form.

(a) Write a program that computes the numerical solution of (3.21). Test your program by choosing the initial temperature $T_0 = 100$°C, $T_s = 0$°C, $r = 1$, and $\Delta t = 0.1$.

(b) Model the cooling of a cup of coffee by choosing $r = 0.03$. What are the units of r? Plot the temperature T as a function of the time using $T_0 = 87$ °C and $T_s = 17$ °C. Make sure that your value of Δt is sufficiently small so that it does not affect your results. What is the appropriate unit of time in this case?

(c) Suppose that the initial temperature of a cup of coffee is 87°C, but the coffee can be sipped comfortably only when its temperature is ≤ 75°C. Assume that the addition of cream cools the coffee by 5°C. If you are in a hurry and want to wait the shortest possible time, should the cream be added first and the coffee be allowed to cool, or should you wait until the coffee has cooled to 80°C before adding the cream? Use your program to "simulate" these two cases. Choose $r = 0.03$ and $T_s = 17$°C. What is the appropriate unit of time in this case? Assume that the value of r does not change when the cream is added. ■

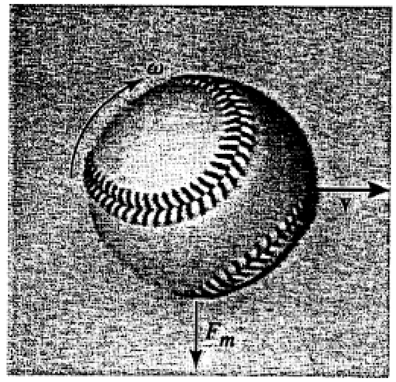

Figure 3.5 The Magnus force on a spinning ball pushes a ball with topspin down.

given by

$$F_{\text{magnus}} \sim v\Delta v. \tag{3.22}$$

We can express the velocity difference in terms of the ball's angular velocity and radius and write

$$F_{\text{magnus}} \sim vr\omega. \tag{3.23}$$

The direction of the Magnus force is perpendicular to both the velocity and the rotation axis. For example, if we observe a ball moving to the right and rotating clockwise (that is, with topspin), then the velocity of the ball's surface relative to the air at the top, $v + \omega r$, is higher than the velocity at the bottom, $v - \omega r$. Because the larger velocity will produce a larger force, the Magnus effect will contribute a force in the downward direction. These considerations suggest that the Magnus force can be expressed as a vector product:

$$F_{\text{magnus}}/m = C_M(\boldsymbol{\omega} \times \mathbf{v}), \tag{3.24}$$

where m is the mass of the ball. The constant C_M depends on the radius of the ball, the viscosity of air, and other factors such as the orientation of the stitching. We will assume that the ball is rotating fast enough so that it can be modeled using an average value. (If the ball does not rotate, the pitcher has thrown a knuckleball.) The total force on the baseball is given by

$$\mathbf{F}/m = \mathbf{g} - C_D|\mathbf{v}|\mathbf{v} + C_M(\boldsymbol{\omega} \times \mathbf{v}). \tag{3.25}$$

Equation (3.25) leads to the following rates for the velocity components:

$$\frac{dv_x}{dt} = -C_D v v_x + C_M(\omega_y v_z - \omega_z v_y) \tag{3.26a}$$

$$\frac{dv_y}{dt} = -C_D v v_y + C_M(\omega_z v_x - \omega_x v_z) \tag{3.26b}$$

$$\frac{dv_z}{dt} = -C_D v v_z + C_M(\omega_x v_y - \omega_y v_x) - g, \tag{3.26c}$$

where we will assume that ω is a constant. The rate for each of the three position variables is the corresponding velocity. Typical parameter values for a 149 gram baseball are $C_D = 6 \times 10^{-3}$ and $C_M = 4 \times 10^{-4}$. See the book by Adair for a more complete discussion.

Curveballs

(a) Create a class that implements (3.26). Assume that the initial ball is released at $z = 1.8\,\text{m}$ above and $x = 18\,\text{m}$ from home plate. Set the initial angle above the horizontal and the initial speed using the constructor.

(b) Write a program that plots the vertical and horizontal deflection of the baseball as it travels toward home plate. First set the drag and Magnus forces to zero and test your program using analytical results for a 40 m/s fastball. What initial angle is required for the pitch to pass over home plate at a height of 1.5 m?

(c) Add the drag force with $C_D = 6 \times 10^{-3}$. What initial angle is required for this pitch to be a strike assuming that the other initial conditions are unchanged? Plot the vertical deflection with and without drag for comparison.

(d) Add topspin to the pitch using a typical spin of $\omega_y = 200\,\text{rad/s}$ and $C_M = 4 \times 10^{-4}$. How much does topspin change the height of the ball as it passes over the plate? What about backspin?

(e) How much does a 35 m/s curveball deflect if it is pitched with an initial spin of 200 rad/s? ■