Modeling & Simulation Warm – up Exercise 1

1. The *composite trapezoidal rule* for numerical integration approximates the integral of a function *f(x)* on a closed interval [*a,b*] divided into *n* equally spaced sections of length Δx as

$$\int_a^b f(x)dx \approx \frac{1}{2}\sum_{i=1}^{n}\{f(x_{i-1})+f(x_i)\}\Delta x = \frac{(b-a)}{2n}\sum_{i=1}^{n}\{f(x_{i-1})+f(x_i)\}.$$

I would like for you to investigate trapezoidal rule integration.

a) Write a program (or modify an existing one) that estimates $\int_0^5 x^2 dx$ using the composite trapezoidal rule for n = 1, 10, 1000 and 10,000.

b) Now modify your program so that you can estimate $\int_0^3 (81-x^4)dx$ for n = 1, 10, 1,000 and 10,000

c) For both (*a*) and (*b*) compare your estimates for different *n* with the exact values of the integral. Please explain why the difference is negative in one case and positive in the other.

2. Please now modify your program from (1) so that you now estimate $\int_0^5 e^{-x^2} dx$ using the composite trapezoidal rule. Since there is no closed form solution for the integral, I would like for you to decide on the value of *n* required to make your answer "reasonable". Note: different students may have different answers for what *n* should be. I'd like for you to develop your own requirements for "reasonable" but will be glad to offer suggestions!

3. If you get through with 1 and 2 in time here is a last warm-up task. Consider the Taylor expansion for the exponential: $e^{-x} = 1 - \frac{x}{1!} + \frac{x^2}{2!} - \frac{x^3}{3!} + \cdots\ ,\ -\infty < x < \infty.$
Write or modify a C++ program that calculates the number of terms required for the expansion to come within 10% of the exact value of the exponential for $x = 0$, $x = 0.01$ x=0.1 and x=1.