Computational Modeling of Pool Games: Sensitivity of Outcomes to Initial Conditions

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ABSTRACT

We present a study of the sensitivity of trajectories of pool balls to initial conditions. In the first component of the study our simulations include all sixteen balls. Variables include cue ball initial velocity and position on the "table". We find that in a certain regime of initial conditions the system seems to show self-similarity, but as the range of initial cue ball angle and initial velocity is restricted, the system exhibits an interesting evolution towards a single point in parameter space, with the ball landing in only one pocket. We also examine the effects of varying the number of balls on the table, and how their dynamics may be interpreted using various plots and maps. Finally, the trajectory of a single cue ball is examined while it moves through the table space. Starting with the cue ball placed in the middle of the right wall of the table (traditional and rectangular in shape) and fired directly downward the system exhibits a two-cycle pattern. Then as the angle of fire is increased the system exhibits a four cycle, a three cycle and finally a two cycle all separated by noisy patterns. Effects of numerical artificialities are briefly discussed.

I. INTRODUCTION

Chaos is derived from the Greek word for "abyss" and may be translated to mean "where chance is supreme". In fact, popular notions equate chaos with randomness, which is not at all the true physical and mathematical definition. Chaos (often "deterministic chaos") is the physical manifestation of sensitivity of a particular set of system outcomes to another set of its initial conditions[1]. Since the introduction of chaos into the field of physics many systems have been studied in order to understand disorder: e.g., cloud formation, weather patterns, and the stock market. The game of pool has also been studied on a number of levels for geometrical, physical, and chaos-related reasons. There are a wide variety of pool simulators[2-5] whose chief purpose is to entertain. Such simulators strive, with varying degrees of accuracy, to be as realistic as possible, including the effects of friction, spin, and non-ideal ball and bumper collisions. Although intricately programmed, enjoyable and in many cases customizable, such simulators lend themselves quite poorly to innovative scientific study due to the inability of a user to vary the model parameters and adjust the numerical output in novel ways. There are algorithms[6,7] which model the behavior of one ball on tables whose boundaries have varying geometries and, in both visual and numerical settings, follow the balls through phase space and track the divergence of paths for initial conditions that could, in principle, start out arbitrarily close.

The purpose of this study is to formulate a reasonably realistic model of pool balls on a typical table and to have the ability to study the sensitivity of the system to initial conditions over a wide range of system parameters. By utilizing different output representations we may easily vary the algorithm to incorporate many different scenarios. The task at hand is largely theoretical, while relying on experimental results for correspondence with actual data. Since much of the results are found by computation, we are also concerned with how the results depend on numerical