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Transient-state groundwater flow in various geometries with wells at arbitrary positions: analytical methods

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Abstract:

We have developed a method for analytically solving the porous medium flow equation in many different geometries for horizontal (two-dimensional), homogeneous and isotropic aquifers containing impermeable boundaries and any number of pumping or injection wells located at arbitrary positions within the system. Solutions and results are presented for rectangular and circular aquifers but the method presented here is easily extendible to many geometries. Results are also presented for systems where constant head boundary conditions can be emulated internal to the aquifer boundary. Recommendations for extensions of the present work are briefly discussed. Copyright © 2003 John Wiley & Sons, Ltd.

KEY WORDS Transient state; analytical method; many wells; various geometries; rectangular aquifers; circular aquifers; impermeable boundaries; computer models

INTRODUCTION

The topic of analytical modeling of groundwater flow has been thoroughly explored and boasts copious appearances in both journals and textbooks (Muskat, 1937; Chow, 1964; Matthews and Russell, 1967; Freeze and Cherry, 1979; Şen, 1995; Kresic, 1997; Serrano, 1997; Hornberger *et al*., 1998; Criss and Phillips, 2001). The bulk of examples covered deal with symmetric flow in rectangular or circular aquifers having wells in the geometric centre of the system. In addition, however, there are some insightful analytical treatments for flow involving arrays of wells with certain symmetries and even flow in a confinement with a circular barrier and an off-centre well (Muskat, 1937). Even so, researchers still resort to numerical techniques (McCuen and Snyder, 1986; Hoggan, 1989) in cases involving either off-centre wells or systems with non-standard geometries. There exist many excellent numerical fluid flow modelling packages that are soundly validated and useful in a wide arena of situations, especially in the cases of underground water (USGS, 2001) and oil (Fluent, 2002). However, analytical modelling, when realizable, can provide deep insight into the physics of a particular system while affording an appreciation for the beauty of the symmetry it possesses. (The literature mentioned here is not exhaustive but representative of the current state of knowledge. Most citations in this paper have robust reference lists (up to hundreds of references) themselves.).

The primary purpose of the work presented here is to provide analytical solutions for horizontal groundwater flow in homogeneous, isotropic systems applicable to a vast variety of geometries and with various types of sources and/or sinks interior to the boundary. Although in our work the barriers are impermeable we will present methods for modelling constant-head systems as well. Secondarily, analytical models can also serve as effective tools for undergraduate student mentorship and research involvement.

The mathematics to be presented here may reasonably represent two types of physical systems. The first is a confined aquifer with line-source wells having constant pumping rates and pumping lengths. The second

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